1. There are two equations that FM proposes and claims proofs by theoretical and empirical means. Both are incorrect from both points of view. The equation equating the absorption of terrestrial radiation to the downward radiation emitted by the atmosphere has no basis in theory and is demonstrably untrue observationally. The equation relating OLR to one half of the terrestrial emission flux density has no theoretical basis and is again demonstrably untrue observationally. Both equations are proposed by FM to fit in with his eventual conclusions.
2. There is confusion about the use of the term optical depth. The proper definition of optical depth is τ = –ln T where T is the transmission for a spectral line of a *specified frequency* through a medium. It refers to the Beer’s Law relationship and can be estimated for each and every line in the terrestrial spectrum, but it is not a simple matter for something as large as the atmosphere. The mean value of such a quantity over the terrestrial spectrum or any other spectrum can only be used as a non-quantitative indication of the general absorption property of the sample substance, the atmosphere in FM’s case.

When applied to the atmosphere, with its ever decreasing pressure as altitude increases and the peculiar temperature variation, the optical depth, *u*, is given by the integral of product of the absorption coefficient and the density of the absorbing substance with a vertical optical path element:

The transmissivity, τ, for any frequency is given by:

The optical depth for any particular frequency is not easily computed since *k* varies with pressure and temperature. Such variations are contained within the HITRAN database.

The above relates just to an optical path normal to the surface and can be modified to take into account off-normal paths, but the above definition suffices for the present argument. There are two methods used by FM to calculate τ. One is to regard the overall transmission as ST and to compare this with the total surface flux density, SU. Using the Kiehl/Trenberth values this gives τ = ln (390/40) = 2.28. This compares the flux density of radiation escaping through the ‘window’ from 750-1250 cm-1 with that emitted by the surface over the 0-1600 cm-1 range. The two flux densities are interdependent in that the more opaque the atmosphere becomes, the smaller the value of ST will become and vice versa, but to convert the two flux densities into a really meaningful property such as the optical depth is not legitimate. FM claims that this particular property is a constant, but that is impossible. The addition to the atmosphere of more CO2 will cause the ‘window’ to become narrower and will reduce the value of ST. The SU/ST ratio would only be constant if the addition of the CO2 reduced the value of SU by an identical fraction, but the addition is supposed to cause an increase in surface temperature and that would increase the value of SU.

The transmission of any particular frequency through the atmosphere is governed by the Schwarzchild equation that takes into account the Beer’s Law reduction in flux density with altitude *and* the enhancement of flux density with altitude from thermal radiation. Thus, for any particular frequency there is a ratio for the flux density transmitted by the atmosphere and including the thermal emission and the flux density emitted by the surface that may be transformed into an optical depth, τS [S for Schwarzchild] that may be estimated using the MODTRAN facility. This shows that the mean value of τS is around 0.54, including the ‘window’ region. FM equates this kind of τS with that given by τ = ln (SU/ST) and therein lies an error that destroys his paper.



There’s nothing amiss with the FM mathematics on his slide shown above. The error is to ignore the computed *TA* and use the ST/SU estimate instead.

Optical density or optical thickness?

The two terms are used interchangeably by some authors, but they can be reserved for the property of individual specific frequencies [optical depth] and the integrated values of optical depth over a range of frequencies [optical thickness]. In neither case are the values inclusive of thermal emission terms. Any calculations using FM’s or K/T’s values for flux densities include the thermal emission terms. In these terms FM’s τA values are given by –ln(ST/SU) = 2.28 are estimates of optical thickness. FM’s calculation of *T*A in the above slide extract includes the optical depths for each frequency in the range, modified by the *B* term that accounts for thermal emission, all integrated over the frequency range, the answer not being given. This is optical thickness offset by thermal emissions and in FM’s terms, using the K/T values would be given by ln[390/235] leading to an overall optical thickness of 0.51, very close to the mean produced by MODTRAN.