Deriving AR1 Autocorrelation Coefficients from Tree-Ring Data.

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May 11, 2006

We assume the proxy records to be closely approximated by random 'noise' superimposed on a slow fluctuating signal component with comparatively large excursions over multi-decadal periods. Below is a 'differencing' technique used to remove the large variance highly correlated slow component from consideration prior to determining the AR1 autocorrelation component.

Consider a proxy-site of N individual tree-ring records each of T years duration. The recorded growth-amplitudes are $X_i(j)$ where i the record number goes from 1 to N and j the index of year goes from 1 to T.

Consistent with the above approximation we define

$$X_i(j) = s_i(j) + \epsilon_i(j)$$

where $s_i(j)$ is the slow component and $\epsilon_i(j)$ is the 'noise' amplitude.

For n small, by definition

$$s_i(j) \approx s_i(j+n)$$

Then the $Y_i(j)$, where $Y_i(j) = X_i(j) - X_i(j+1)$, are

$$Y_i(j) = \epsilon_i(j) - \epsilon_i(j+1)$$

and are independent of the signal.

By definition averaging over years j where the time span T is large,

$$< \epsilon_i(j)^2 >_j = \sigma_i^2$$

where the σ_i is the standard deviation of the 'noise' for proxy i.

If the noise is assumed to be represented by an AR1 process

$$< \epsilon_i(j) \times \epsilon_i(j+n) >_j = \alpha_i^n \sigma_i^2$$

where for each record i the α_i are the annual AR1 decrement factor and the σ_i are the standard deviations.

Using the above definitions and averaging over j gives

$$< Y_i(j) Y_i(j+1) >_j = -(1-\alpha_i)^2 \sigma_i^2$$

and

$$\frac{1}{2} < Y_i(j).Y_i(j) >_j = (1-\alpha_i)\sigma_i^2$$

Using the above two equations we find

$$\alpha_i = 1 + 2 \frac{< Y_i(j) Y_i(j+1) >_j}{< Y_i(j).Y_i(j) >_j}$$

Finally the mean α_0 for the full set of N proxies is

$$\alpha_0 = < \alpha_i >_i$$

As a representative example, we applied the above results to an ensemble of seventy proxy records of the North American network spanning the years 1400 to 1980 and consisting of a mix of bristle-cone-pines and others. To avoid errors that could be potentially be introduced by fast signal changes over the last century, we confined our averages to the time span 1400-1880. Application of the above gave a mean annual AR1 decrement factor close to 0.15 ± 0.03, where the quoted error is the internal consistency result derived from the ensemble of records. In view of the approximations involved in this procedure we estimate that the actual error could be underestimated and conservatively perhaps be as large as ± 0.06. This still constitutes, at most, a minor reddening of the noise spectrum.